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Reliability allocation under uncertainty with optimism & pessimism

Aniruddha Samanta[1], Sanchita Sarkar[2], Angshu Kumar Sinha[1], Srabani Guria Das[1]
[1] Assistant Professor, Department of Basic Science (Mathematics), NSHM Knowledge Campus, Durgapur, West Bengal, India
[2] Professor, Department of Basic Science (Mathematics), NSHM Knowledge Campus, Durgapur, West Bengal, India

Abstract
A crucial issue in the early stages of a system's design and development is reliability allocation. To reach the desired reliability, a variety of reliability allocation strategies are applied. Among them, feasibility of objectives (FOO) technique is frequently employed for system reliability allocation. However, this method has two major drawbacks. The measurement scale and the order weight of the reliability allocation factors is not taken into account is this method.
Another important factor of decision making is the consideration of different mind-set of decision makers. Practically, decision makers in multi-criteria decision making (MCDM) are not in the same mind-set level.
Hence in order to overcome the drawbacks of the FOO strategy, a reliability allocation method is presented here with addressing the decision makers' mind-set differences. Moreover, the fuzziness, minimal variance OWGA weights have been incorporated to perform more practical reliability allocation. To confirm the effectiveness of the suggested strategy, a case study on reliability allocation in an airborne radar system is also taken into consideration. Ultimately, the outcomes are computed from several optimistic and pessimistic perspectives and compared with FOO methodology. This comparison demonstrates the benefits and superiority of the suggested method.

Keywords: Reliability allocation, FOO technique, fuzziness, optimistic & pessimistic decision making

Introduction
Assigning reliability is an essential responsibility at manufacturing of any engineering system. Assuring that the system can accomplish its target goals and allocating the target reliability to the subsystems are the primary objectives (Bona *et al*., 2018 [2], Ronchieri and Canaparo, 2018) [15]. When it comes to reliability allocation, how resources are allocated has a big impact on the system's operational success and product quality. Consequently, in order to assign reliability to the system, an efficient reliability allocation technique is needed.
In 1957 Advisory Group on Reliability of Electronic Equipment (AGREE, 1957) [1] suggested a way of allocating reliability that took into account the complexity and criticality of the subsystem rather than the failure rate. Following that, in 1964, Aeronautical Radio Inc. published the ARNIC approach in light of the subsystem failure rate (William and Alven, 1964) [17]. Brcha (Brcha, 1964) put out a different allocation strategy that year, taking into account the state-of-the-art, complexity, operational time and environmental conditions. On the other hand, Karmiol presented the allocation method based on operational time, complexity, criticality, and state-of-the-art in 1965 [9]. More recently, the Mil-hdbk-338B manual (United States Department of Defence, 1988) for military reliability design published the feasibility of objectives (FOO) technique, which is another significant technique. Additionally, there are ways for allocating reliability such as improved agree approach (Liang *et al*., 2018) [2], integrated factor method (Felice *et al*., 2010) [7] and average weighting allocation (Kuo, 1999) [10]. However, every one of them has a flaw in the measurement scale. Discrete ordinal scales are employed in these procedures to measure the ratings of the components that cause the results to be misleading. Additionally, there are differences in the weighting of the system components, which complicates decision-making. Chang *et al*. (2009) [4] created a few techniques based on the Yager's OWA operator to address these issues. In order to address the aforementioned issues, Chen (2009, 2015) [5, 6] also suggested a reliability allocation technique that takes uncertain preferences into account.
When making decisions, especially in situations involving uncertainty or risk, people often approach the process from either an optimistic or pessimistic perspective. (Sahoo *et al*., 2023; Maitra *et al*. 2023) [13, 16] These perspectives influence how they evaluate potential outcomes and choose among alternatives. So incorporation of optimistic and pessimistic perspective in decision making is anticipated as very important.
Here a maximal entropy minimal variance ordered weight averaging (MEMV-OWA) operator is used in this study. An effort has been made to maximize entropy and minimize the variance of the weighting vector using this operator. Furthermore, our suggested approach helps get around the shortcomings of the earlier reliability allocation techniques & taking decisions in optimistic & pessimistic perspective.
The remaining section of this article is organized as follows: A survey on reliability allocation technique and its shortcomings is created in section 2. A theoretical overview of the ordered weight averaging operator and its characteristics is given in Section 3. Section 4 proposes a reliability allocation procedure based on this idea. In section 5, a numerical example has been solved to demonstrate the adaptability of the suggested methodology. In the same section, there is also a comparison with the FOO technique and a discussion under different optimistic and pessimistic perspectives. Section 6 offers a final assessment of the entire work.

Reliability Allocation Methods
Reliability allocation is an essential process for allocating the desired reliability into each subsystem. Several

reliability allocation strategies exist, such as average weighted allocation method (Kuo, 1999) [10], FOO technique (MIL-HDBK-338B, 1988) [14], AGREE (1957) [1], and ARNIC (1964) etc. The FOO methodology is one of the most useful methods for reliability allocation among these. In this method allocation factors are system intricacy, state-of-art, performance time and environmental conditions. The specialists evaluate these factors numerically, drawing on their prior experience. Every rating is on a scale from 1 to 10, with the values assigned as follows:

System Intricacy (I): A value of 1 is assigned to the least intricate system, and a value of 10 is assigned to the most intricate system.

State-of-Art (S): One represents the highly developed design, while ten represents the least developed.

Performance Time (P): The subsystem that operates for the shortest amount of time in between the completion time is allocated as 1 and the subsystem that operates for the entire completion period is assigned as 10.

Environment (E): Components expected to face harsh and grave circumstances are given a value of 10, while those expected to face the least severe circumstances are given a value of 1.

The design professionals assign scores based on their expertise and level of understanding in triangular fuzzy number. The four ratings for each subsystem are defuzzified and multiplied to get the total ratings. That is, ISPE= I×S×P×E. where I, S, P and E stand for system complexity, state of the art, performance time, and environment parameters respectively.

Assume that N subsystems make up a system. Let C'_K represents the complexity of subsystem k, R'_K is the composite rating for subsystem k, λ_S is the system failure rate and λ_k is the failure rate assigned to the kth subsystem. The composite ratings are added together to form R' and r'_{ik} is the i^{th} rating for each factor in subsystem k for all i € (I, S, P, E). T is the total completion time, while R_s is the target reliability. The fundamental equations of FOO technique (MIL-HDBK-338B,1988) [14] are:

$$\lambda_S = \sum \lambda_k \qquad (1)$$

$$\lambda_k = C'_K \lambda_S \qquad (2)$$

$$C'_K = \frac{R'_k}{R'} \text{ for all k} \qquad (3)$$

$$R'_k = r'_{Ik} \times r'_{Sk} \times r'_{Pk} \times r'_{Ek} \qquad (4)$$

$$R' = \sum_{K=1}^{N} R'_k \qquad (5)$$

Drawbacks of FOO techniques (Chang *et al*.,2009) [4]

There are many uses for the FOO technique in reliability allocation. Nonetheless, there are two significant flaws in this technique that have drawn criticism. The discrete ordinal measurement scale used to evaluate the four factors (I, S, P, and E) is the first shortcoming. Multiplication is meaningless as the ISPE value is determined by multiplying the ratings of I, S, P, and E. The second shortcoming is that the components are not equally weighted, which makes it difficult to analyse and comprehend the results. If two components have ISPE values of 9×2s×2×2=72 and 7×3×2×2=84 respectively, the former should be assigned a higher reliability than the latter. In actuality, however, the FOO technique assigns a lower reliability to the earlier because the earlier ISPE value was lower than the later. As a result, the weights of I, S, P, and E are not equal to one another.

Theoretical Background

Definition 1. Triangular fuzzy number (Kar *et al*., 2019) [8]

It is a fuzzy number represented with three points as $A = (a, b, c)$ and the membership function is given as:

$$\widetilde{\mu_A} = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \le x \le b \\ \frac{C-x}{C-b}, & b \le x \le C \\ 0, & x > C \end{cases}$$

Definition 2. OWA operator (Chen *et al*., 2015) [6]

An n-dimensional OWA operator is a mapping F: $R^n \rightarrow R$, with an associated weight vector $w=(w_1; w_2;\ldots; w_n)^T$ such that $\sum w_i = 1$; $\forall\, w_i$€ [0;1]; i = 1;2;………,n: with the properties,

$$f(a_1,a_2,\ldots,a_n) = \sum^{n}{}_{i}=1 w_i\, b_i$$

where b_i is the i^{th} largest element in the vector $(a_1, a_2, \ldots, a_n)$ and $b_1 \le b_2 \ldots \le b_n$

Definition 3. Measure of Orness (Chen *et al*., 2015) [6]

Let F is an OWA operator with a weight function w= $(w_1, w_2,\ldots, w_n)$. The degree of orness associated with the weight vector of the operator is defined as:

$$\text{Orness (W)} = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)$$

where orness (W) € [0,1] is denoted by a, which is a situation parameter.

a = 1 describes the situation when the decision maker is maximally optimistic (a pure optimistic) and a = 0.5 is considered when the decision maker faces a moderate assessment.

Definition 4. Measure of Entropy (Chen *et al*., 2015; Liaw *et al*., 2011) [6, 12]

The measure of entropy is the degree of utilization of information in an uncertain environment. This is also known as the measure of dispersion and is defined by

$$\text{Disp (W)} = -\sum_{i=1}^{n} w_i \ln w_i$$

Properties:

1. when w_i = 1 the dispersion of W is minimum, and disp(W)= 0.($\sum w_i$ = 1) This denotes that only one criteria is taken in the aggregation process.

2. when $w_i = \frac{1}{n}$, i=1,....,n, the dispersion of W is maximum and disp(W)=ln(n). This denotes that all criteria are taken in the aggregation process.

Definition 5. Measure of Variance (Chen *et al*., 2015) [6]
The measure of variance deduces the variability of weight vector for a given level of orness and is explained as

$$D^2(w) = \frac{1}{n}\sum_{i=1}^{n} w_i^2 - \frac{1}{n^2}$$

Basically, measure of variance helps to avoid overestimating of a single attribute and control the variance of weighting vector in the decision-making process including multiple criteria.

Definition 6. Maximum Entropy-Minimum Variance Ordered Weight Averaging Operator (MEMVOWA):
Depending upon the concept of entropy and variance of OWA operator, a bi-objective model can be formulated to obtain the weighting vector of the MEMV-OWA operator (Chen *et al*., 2015) [6]. Here entropy is maximized to utilize the uncertain information of the decision maker's experience and variance of the weighting vector is minimized to avoid overestimation of the decision maker's preferences. So, the model is:

$$\text{Maximize} : -\sum_{i=1}^{n} w_i ln w_i$$

$$\text{Minimize} : \frac{1}{n}\sum_{i=1}^{n} w_i^2 - \frac{1}{n^2}$$

$$\text{Subject to: } \sum_{i=1}^{n} \frac{n-i}{n-1} w_i = \alpha, (0 \le \alpha \le 1).$$

$$\sum_{i=1}^{n} w_i = 1, 0 \le w_i \le 1, i = 1, 2, \ldots\ldots, n.$$

where, w_i denotes weight of the i^{th} criteria, n denotes number of attributes, a denotes situation parameter. For solving this model, weighted sum method is used here to transform the bi-objective problem into a single objective model giving equal weights to both the objectives. Then LINGO software is used for final solution.

Proposed Reliability Allocation Method:
The fundamental steps of allocation method are:
Step 1: Consider the subsystems that make up the system.
Step 2: Decide the system's overall completion time and target reliability.
Step 3: Specify the scale for every criterion. The values are considered as a triangular fuzzy numerical rating.
Step 4: Create a matrix taking into account the ratings in triangular fuzzy form and then defuzzified & normalize it.
Step 5: The rating for k^{th} subsystem will be calculated by $R_k^/ = \prod_{i=1}^{n} (r_{Ik}^/)^{w_i}$, where w_i will be decided by MEMV-OWA operator.
Step 6: Using equation $R^/ = \sum_{K=1}^{N} R_k^/$, determine the overall rating $R^/$ for the system.
Step 7: Using the formula $C_k^/ = \frac{R_k^/}{R^/}$, determine the complexity C'_k for k^{th} subsystem for all k.
Step 8: Utilizing the equation $\lambda_S = -\frac{\ln (R_S)}{T}$, determine the system failure rate.
Step 9: Apply equation 2 to determine the allocated failure rate for every subsystem.
Step 10: Compute the reliability allocation values at various *a* as well as the overall ratings.
Step 11: Examine the overall outcomes and assign each subsystem the optimal level of reliability.

Case Study
A case study of an airborne radar system based on Chen and Wang (2009) [5] example, is examined here to establish proposed approach's applicability. This radar system, designed to detect objects by measuring their distance, angle or velocity through radio waves. It has various uses: tracking weather patterns, identifying guided missiles, or locating aircraft and spacecraft. The system's core consists of an equipment rack, accessory installation materials and five principal line reducible units (LRUs): Radar data computer (RDC), radar target data processor (RTDP), transmitter (TRAN), antenna (ANT), and filter.
In this system, an antenna (ANT) is utilized for both transmitting and receiving the signal, while a transmitter (TRAN) generates electromagnetic waves in the radio or microwave domain. Subsequently, the FILTER assists with signal modulation and filtering, and at last the processor (RTDP) determines the attributes of the objects. Additionally, one of the main subsystem of airborne radar is the RDC (Radar Data Computer) which manages radar functions such target detection, tracking, pre-processing, radar control and post-processing. Figure 1 illustrates the overall structure of the airborne radar system.

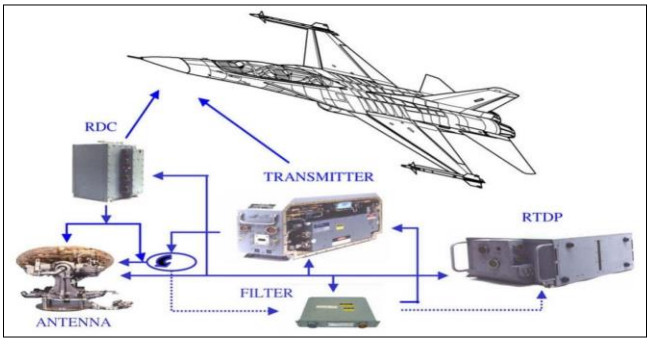

Fig 1: General structure of the airborne radar system

1. Results

Solution by FOO technique
The ratings of the allocation factors (I, S, P and E) of FOO technique are shown in table 1. The TFN value is defuzzified using averaging technique. System reliability target & completion time are set as 0.997 & 2.4h respectively. Then,

$$\lambda_S = -\frac{\ln (R_S)}{T} = -\frac{\ln (0.99)}{2.4} = 0.00125187$$

Table 1: Numerical rating for reliability allocation

Subsystem	Intricacy(I)	State-of-art(S)	Performance time(P)	Environment (E)
RTDP	[7,8,9]	[8,9,10]	[7,8,9]	[6,7,8]
RDC	[5,6,7]	[5,6,7]	[7,8,9]	[4,5,6]
TRAN	[7,8,9]	[7,8,9]	[7,8,9]	[8,9,10]
ANT	[3,4,5]	[4,5,6]	[4,5,6]	[1,2,3]
FILTER	[1,2,3]	[1,2,3]	[2,3,4]	[1,2,3]

Overall rating of RTDP is 8×9×8×7=4032, Complexity is $C'_k = \frac{R'_k}{R'} = \frac{4032}{10304} = 0.391$, allocated failure rate is 125.187×0.391= 48.948/10^5h. Similarly, the failure rate for other subsystem is shown in table 2.

Table 2: Allocation results by FOO technique

Subsystem	R'_k	C'_k	Allocated failure rate (per 10^5h)
RTDP	4032	0.391	48.948
RDC	1440	0.139	17.4009
TRAN	4608	0.447	55.959
ANT	200	0.019	2.378
FILTER	24	0.0023	0.287

Table 3: Reliability allocation by proposed method at *a*=0.1(strongly pessimistic)

Subsystem	V(k)	C'_k	Allocated failure rate	Reliability
RTDP	1.198062408	0.346564159	40.13394561	0.999136412
RDC	0.699130203	0.189232219	23.37320689	0.999542154
TRAN	1.083437189	0.314153247	36.25910727	0.999231193
ANT	0.41946226	0.12588867	13.72415833	0.999861614
FILTER	0.161125812	0.05829382	5.845646138	0.999462217

Table 4: Reliability allocation by proposed method at *a*=0.3(moderately pessimistic)

Subsystem	V(k)	C'_k	Allocated failure rate	Reliability
RTDP	1.187312693	0.344956107	39.94357989	0.999052897
RDC	0.68415875	0.194473734	23.31449135	0.999451612
TRAN	1.074199938	0.315323474	36.41079362	0.999137756
ANT	0.42287463	0.115285268	13.96441133	0.999677783
FILTER	0.180294417	0.057971318	5.816926392	0.99976398

Table 5: Reliability allocation by proposed method at *a*=0.5(moderate assessment)

Subsystem	V(k)	C'_k	Allocated failure rate	Reliability
RTDP	1.191560872	0.343386745	39.73517375	0.999057507
RDC	0.689997578	0.196742529	23.33622108	0.999450761
TRAN	1.095863493	0.316572063	36.54081563	0.999134075
ANT	0.417326832	0.126586574	13.88826787	0.999676938
FILTER	0.171471223	0.048733287	5.680743593	0.999873525

Table 6: Reliability allocation by proposed method at *a*=0.7(moderately optimistic)

Subsystem	V(k)	C'_k	Allocated failure rate	Reliability
RTDP	1.192790257	0.332815253	39.54900251	0.99915198
RDC	0.723935118	0.194989067	23.36769827	0.999450031
TRAN	1.104625254	0.308825183	36.69793147	0.999220531
ANT	0.429759332	0.117867519	13.94271683	0.999676109
FILTER	0.171628724	0.048504979	5.673672598	0.99986825

Table 7: Reliability allocation by proposed method at *a*=0.9(strongly optimistic)

Subsystem	V(k)	C'_k	Allocated failure rate	Reliability
RTDP	1.194039566	0.330372428	39.37594834	0.999045372
RDC	0.709875764	0.194250895	23.39682874	0.999539431
TRAN	1.126487912	0.308064455	36.84645069	0.999217112
ANT	0.424191814	0.118152174	13.96576305	0.999564296
FILTER	0.170895237	0.047383252	5.635978862	0.999765908

2. Comparison and discussion

Table 8 and Figure 2 present a comparison of the simulation results in order to highlight the flexibility of the suggested strategy.

Table 8: Comparison of allocated failure rate to the subsystems using two methods

Subsystem	Proposed method					FOO technique
	a = 0.1	*a* = 0.3	*a* = 0.5	*a* = 0.7	*a* = 0.9	
RTDP	40.13394561	39.94357989	39.73517375	39.54900251	39.37594834	48.948
RDC	23.37320689	23.31449135	23.33622108	23.36769827	23.39682874	17.4009
TRAN	36.25910727	36.41079362	36.54081563	36.69793147	36.84645069	55.959
ANT	13.72415833	13.96441133	13.88826787	13.94271683	13.96576305	2.378
FILTER	5.845646138	5.816926392	5.680743593	5.673672598	5.635978862	0.287

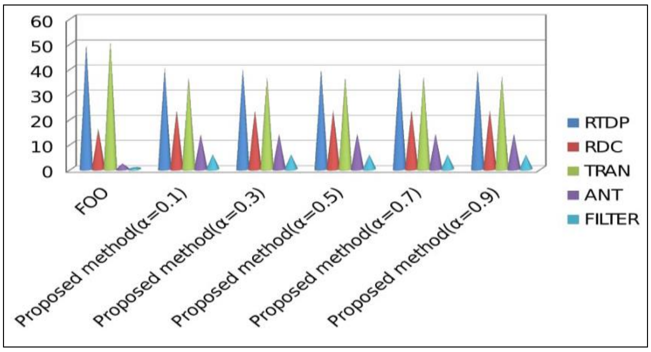

Fig 2: Comparison of allocated failure rate using two methods

According to Table 2's results using the FOO approach, RTDP and TRAN had higher assigned failure rates than ANT and FILTER. This method assigns a very low failure rate(0.287/10^5h) to the filter. In actuality, building a subsystem with such a low failure rate would require destroying a significant amount of resources, which could have an impact on the budget. However, the failure rate assigned to the filter by our suggested method is minimal, 5.680743593/10^5h (at *a* = 0.5). Hence, a subsystem with this failure rate can be created with noticeably fewer resources than those needed for the FOO technique. Additionally, according to the FOO approach, the assigned failure rate of TRAN and RTDP suggests a high failure rate, which causes frequent failures and expensive subsystem repair. However, compared to the failure rate achieved using the FOO technique, the failure rates acquired using the proposed method for RTDP and TRAN are comparatively lower.

Thus, based on all of these evaluations and data, we can say that the suggested approach assigns the failure rate to the subsystems in a more realistic manner.

The suggested approach has the following benefits:

1. **Advantage in measuring scale:** The multiplication and division operations in the FOO approach have no value because the ordinal scale measurement is useless. In our suggested approach we assume an equal numerical rating scale and normalize the matrix. As a result, the outcomes of this approach make sense.

2. **Benefit of ordered weight consideration:** Unlike the FOO technique, the suggested method is able to evaluate the ordered weight of elements. Given that the elements' ordered weight is a crucial attribute in reliability allocation, the suggested approach is more beneficial for making decisions.

3. Last but not least, this approach can handle scenario parameters values and fuzziness that can show the decision maker's accurate level of optimism (strictly optimistic, moderate assessment, or pessimistic) to flexibly aggregate the factor values.

Table 9 provides a tabular summary of the comparison between the benefits of the FOO methodology and the suggested method. '√' indicates that the associated attribute is taken into consideration, while '×' indicates that it is not.

Table 9: Comparison of proposed method and FOO technique

Special feature				
	Measurement scale	**Order weight**	**Situation parameter**	**Fuzziness**
Proposed Technique	√	√	√	√
FOO technique	×	×	×	×

Conclusion

This study is aimed to overcome the measurement scale & order weight deficiencies of FOO technique. Also it focuses on the optimistic & pessimistic perspective of decision makers. The suggested reliability allocation method's applicability was explained in detail in this paper. Here, the adaptability of this method is demonstrated with the help of an airborne radar system and a comparison with the FOO technique is also provided. The following are the main advantages of this proposed method: 1) it can handle measurement scale problems and ordered weighted problems effectively; 2) it can cover decision makers' differences & fuzziness efficiently and 3) reliability allocation factors are not limited to I, S, P, and E; it can also take into account other factors like cost, risk, complexity and maintenance.

As a result, this approach can be applied in a variety of domains and sectors, assisting decision-makers in figuring out how best to distribute resources within a system. In this case, the allocation factor rating has been taken into consideration as a triangular fuzzy value. However, experts can provide their opinions as fuzzy information such as very high, high, medium and low. Therefore, a key area for future development with this suggested strategy is the consideration of fuzzy linguistic information.

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